May 2021

UPPER SCHOOL SUMMER MATH PACKET
Incoming 8th Grade: Honors Algebra II

Dear Upper School Students,

This summer, we encourage you to continue to foster a belief in the importance and enjoyment of mathematics at home. Being actively involved in mathematical activities enhances learning.

In preparation for the 2021-2022 school year, each student entering middle school is required to complete a summer math review packet. Each packet focuses on the prerequisite concepts and skills necessary for student success in each math class. The topics within this packet are important foundational concepts. READ THE INSTRUCTIONS. Even if it doesn't say "Show Your Work" at the top of the page, **you are expected to show your work on all pages.** If you need extra space, you must use and attach scratch paper to the back of the packet.

Please bring your completed math packet (with scratch work attached) with you on the first day of school in August. Your math teachers will be collecting them, and the packets will be graded for timeliness and thoroughness of completion.

Have a wonderful summer!

The Middle School Mathematics Department

Solve each equation.

1. $-3x - 9 = -27$	2. $25 + 2(n + 2) = 30$	3. $-9b - 6 = -3b + 48$
4. $5 - (m - 4) = 2m + 3(m - 1)$	5. $-24 - 10k = -8(k + 4) - 2k$	6. $f - (-19) = 11f + 23 - 20f$
7. $\frac{3}{4}d - \frac{1}{2} = \frac{3}{8} + \frac{1}{2}d$	8. $-0.5g + 13 = 3g$	9. $-5(h + 12) - (4h - 2) = h - 8$
10. $\lvert 3x + 4 \rvert = 16$	11. $3\lvert x - 5 \rvert = 27$	12. $-8\lvert 2x - 6 \rvert + 4 = -60$

Solve each word problem algebraically.

13. The sum of two consecutive integers is one less than three times the smaller integer. Find the two integers.	14. The length of a rectangular picture is 5 inches more than three times the width. Find the dimensions of the picture if its perimeter is 74 inches.

Solving Multi-Step Equations

1. Clear parentheses using the distributive property.
2. Combine like terms within each side of the equal sign.
3. Add/subtract terms to both sides of the equation to get the terms with variables on one side and constant terms on the other side.
4. Isolate the variable by multiplying/dividing both sides of the equation by the number with the variable.

Ex: $3(2x - 5) - 3 = 2x + 8 + 6x$

$6x - 15 - 3 = 2x + 8 + 6x$

$6x - 18 = 8x + 8$

$-8 \quad -8$

$6x - 26 = 8x$

$-6x \quad -6x$

$\frac{-26}{2} = \frac{2x}{2}$

$-13 = x \rightarrow \boxed{x = -13}$

Solving Absolute Value Equations

1. Isolate the absolute value.
2. Break the absolute value equation into two separate equations. For the first equation, set the expression inside the absolute value notation equal to the opposite side of the equation. For the second equation, make the number on the opposite side negative.
3. Solve each equation.

Ex: $-3|3x+2| - 2 = -8$

$-3|3x+2| - 2 = -8$

$+2 \quad +2$

$\frac{-3|3x+2|}{-3} = \frac{-6}{-3}$

$|3x+2| = 2$

$3x + 2 = 2 \qquad 3x + 2 = -2$

$x = 0 \qquad x = -\frac{4}{3}$

$\boxed{x = \{0, -\frac{4}{3}\}}$

Solving Word Problems Algebraically

1. Define a variable.
2. Write an equation.
3. Solve the equation.
4. Label your answer with the appropriate units.

Ex: Bobby is 4 years younger than twice Jimmy's age. If Bobby is 26 years old, how old is Jimmy?

Let j = Jimmy's age

$2j - 4 = 26$

$j = 15$

→ **Jimmy is 15 years old**

Solve each inequality. Graph the solution on a number line.

15. $-6x + 3 > -39$	16. $25 - 3(n - 2) \geq -8n + 6$
17. $8g - 6(g + 1) < 4(2g - 9)$	18. $7k + 1 \leq 8$ or $-7 < k - 10$
19. $-4 < 3b + 2 \leq 20$	20. $9 < -3m < 24$
21. $y + (-6) \geq -13$ or $-3y + 8 > -7$	22. $\|2x + 5\| < 13$
23. $7\|w - 6\| \geq 21$	24. $-2\|3m\| + 3 < -51$

Solving & Graphing Inequalities

1. Solve the inequality as if it is an equation.
2. If you multiply or divide both sides of the inequality by a negative number, flip the inequality sign.
3. Write your answer with the variable on the left of the inequality sign.
4. Graph the solution on a number line. Make an open circle on the number if the number is not included in the solution (< or >) and make a closed circle if the number is included (≤ or ≥). Shade to the left for less than (< or ≤) and shade to the right for greater than (> or ≥).

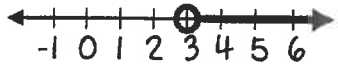
Ex: $-24 > 3x - 6 - 9x$

$-24 > -6x - 6$

$+6 \qquad +6$

$\frac{-18}{-6} > \frac{-6x}{-6}$

$3 < x \rightarrow \boxed{x > 3}$



Compound Inequalities

"Or" Inequalities

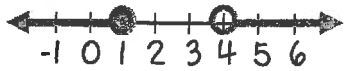
1. Solve each inequality separately and graph the solution to each on one number line.

Ex: $x + 2 > 6$ or $-2x \geq -2$

$x + 2 > 6 \quad$ or $\quad \frac{-2x}{-2} \geq \frac{-2}{-2}$

$-2 \quad -2 \qquad\qquad -2 \quad -2$

$\boxed{x > 4 \quad \text{or} \quad x \leq 1}$



"And" Inequalities:

1. Isolate the variable, making sure to do the same thing to all 3 parts of the inequality.
2. Graph the solution to each part of the compound inequality and see where those graphs overlap. The overlapping part is the solution.

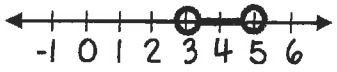
Ex: $3 < 2x - 3 < 7$

$3 < 2x - 3 < 7$

$+3 \qquad +3 \quad +3$

$\frac{6}{2} < \frac{2x}{2} < \frac{10}{2}$

$\boxed{3 < x < 5}$



Absolute Value Inequalities

1. Isolate the absolute value.
2. Change the absolute value inequality into a compound inequality. For > or ≥, turn it into an "or" inequality. For < or ≤, turn it into an "and" inequality. For the first inequality, keep everything the same, except eliminate the absolute value symbols. For the second inequality, make the number on the opposite side negative and flip the inequality sign.
3. Solve and graph the compound inequality.

Ex: $|x + 1| - 3 \geq -1$

$|x + 1| - 3 \geq -1$

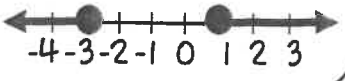
$+3 \qquad +3$

$|x + 1| \geq 2$

$x + 1 \geq 2 \quad$ or $\quad x + 1 \leq -2$

$-1 \quad -1 \qquad\qquad -1 \quad -1$

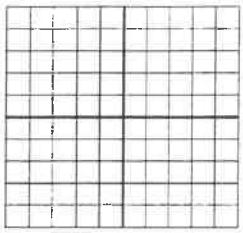
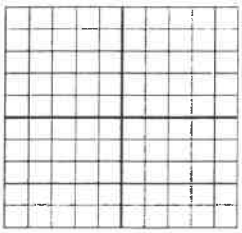
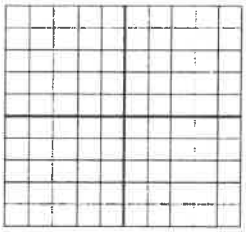
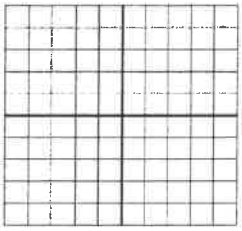
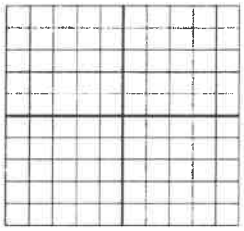
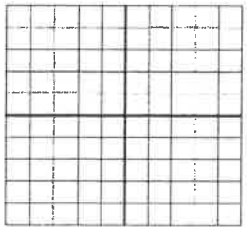
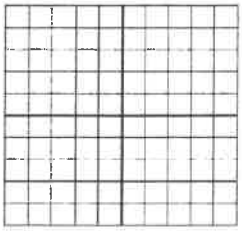
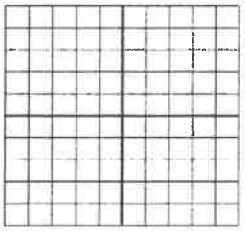
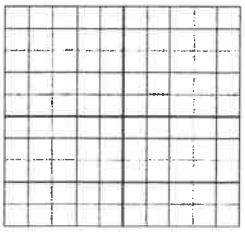
$\boxed{x \geq 1 \quad \text{or} \quad x \leq -3}$



Find the slope of the line that passes through the pair of points.

25. (9, -3) and (9, -8)	26. (-8, 5) and (3, -6)	27. (7, -1) and (15, 9)

Graph each line.

28. $y = -\frac{3}{2}x + 2$ 	29. $y = x - 3$ 	30. $y = \frac{1}{3}x + 5$ 
31. $2x - y = -2$ 	32. $x + y = 4$ 	33. $3x + 4y = -12$ 
34. $y + 3 = \frac{1}{2}(x + 2)$ 	35. $y - 1 = \frac{2}{3}(x - 3)$ 	36. $y - 2 = 0$ 

Write the equation of the line in point-slope, slope-intercept, and standard form.

37. Line passing through point (3, 5) with a slope of 1	38. Line passing through points (-4, 2) and (0, 3)	39. Line passing through points (1, 3) and (2, 5)

Finding Slope from 2 Points

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Special Cases:

$\frac{0}{\#}$ → slope = 0 $\frac{\#}{0}$ → slope is undefined

Ex: Find the slope of the line that passes through the points (-9, -3) and (7, -7)

$m = \frac{-7-(-3)}{7-(-9)} = \frac{-4}{16} = \boxed{-\frac{1}{4}}$

Slope-Intercept Form

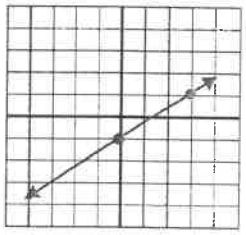
$y = mx + b$

m = slope & b = y-intercept

Graphing from Slope-Intercept Form:

1. Make a point at the y-intercept.
2. Use the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ to make more points.
3. Connect the points to form a line.

Ex: Graph $y = \frac{2}{3}x - 1$



y-intercept is -1
slope = $\frac{2}{3}$, (so from the y-intercept go up 2 & right 3)

Standard Form

$Ax + By = C$

A, B, & C are integers & A is not negative

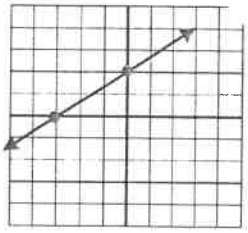
Graphing Using Intercepts:

1. Find the x-intercept by substituting 0 for y.
2. Find the y-intercept by substituting 0 for x.
3. Make a point at each intercept and then connect the points to form a line.

Ex: Graph $2x - 3y = -6$

x-intercept: $2x - 3(0) = -6$
$2x = -6 \rightarrow x = -3$
(-3, 0)

y-intercept: $2(0) - 3y = -6$
$-3y = -6 \rightarrow y = 2$
(0, 2)



Point-Slope Form

$y - y_1 = m(x - x_1)$

m = slope & (x_1, y_1) is a point on the graph

Converting Point-Slope Form to Slope-Intercept Form:

1. Distribute m.
2. Move y_1 to the other side of the equation.

Converting Slope-Intercept Form to Standard Form:

1. Bring the x term to the left.
2. If there are fractions in the equation, multiply everything through by the least common denominator.
3. If A is negative, multiply everything through by -1.

Ex: Write the equation of the line passing through the points (-1, 2) and (3, 4) in point-slope form. Then convert it to slope-intercept and standard form.

$m = \frac{4-2}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$

Point-Slope Form: $\boxed{y - 2 = \frac{1}{2}(x + 1)}$

Convert to Slope-Intercept Form:

→ $y - 2 = \frac{1}{2}x + \frac{1}{2}$ → $\boxed{y = \frac{1}{2}x + \frac{5}{2}}$

Convert to Standard Form:

→ $-2\left(-\frac{1}{2}x + y = \frac{5}{2}\right)$ → $\boxed{x - 2y = -5}$

Determine whether the lines are parallel, perpendicular, or neither. Justify your answer.

40. $y = 2x - 8$ $y = \frac{1}{2}x + 6$	41. $y = x$ $x + y = -2$	42. $3x + 2y = 18$ $y + 4 = -\frac{3}{2}(x - 4)$

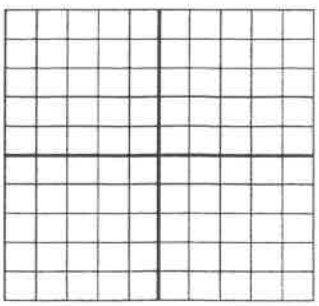
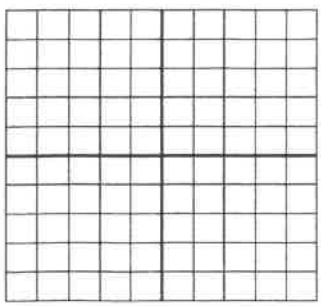
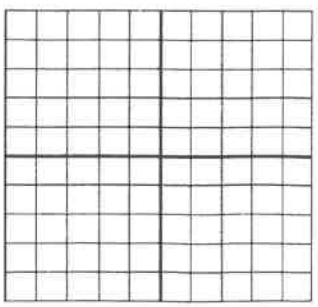
Write the equation of the line parallel to the given line that passes through the given point in slope-intercept form.

43. $y = -4x - 2$; $(0, -1)$	44. $2x - y = -4$; $(2, 5)$

Write the equation of the line perpendicular to the given line that passes through the given point in slope-intercept form.

45. $y = \frac{2}{3}x - 9$; $(-6, -2)$	46. $4x + y = -6$; $(4, 5)$

Graph the solution to each linear inequality.

47. $y \leq -4x - 3$	48. $2x - y < 1$	49. $x + 3y > 3$
		

Parallel & Perpendicular Lines

Parallel Lines have the *same slope* but different y-intercepts.

Perpendicular Lines have *opposite reciprocal slopes.*

Writing Equations of Parallel Lines:

1. Find the slope of the original line by first converting it to slope-intercept form if it is in Standard Form. The slope of the line parallel will have that same slope.
2. Use the given point along with the slope you just found to write the equation of the line in point-slope form.
3. Convert the point-slope form equation to slope-intercept form.

Ex: Write the equation of the line that is parallel to the line $y = 3x - 5$ and passes through the point $(-2, 4)$.

$y = 3x - 5$

$m = 3$, so slope of parallel line is 3, too

$\rightarrow \quad y - 4 = 3(x + 2)$

$\rightarrow \quad y - 4 = 3x + 6$

$\rightarrow \quad \boxed{y = 3x + 10}$

Writing Equations of Perpendicular Lines:

1. Find the slope of the original line. The slope of the line perpendicular will have the opposite (negative) reciprocal slope.
2. Use the given point along with the slope you just found to write the equation of the line in point-slope form.
3. Convert the point-slope form equation to slope-intercept form.

Ex: Write the equation of the line that is perpendicular to the line $x - 3y = -6$ and passes through the point $(-1, 1)$.

$x - 3y = -6 \rightarrow -3y = -x - 6$

$\rightarrow y = \frac{1}{3}x + 2$

$m = \frac{1}{3}$, so slope of perpendicular line is -3

$\rightarrow \quad y - 1 = -3(x + 1)$

$\rightarrow \quad y - 1 = -3x - 3$

$\rightarrow \quad \boxed{y = -3x - 2}$

Linear Inequalities

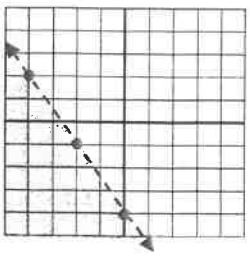
1. Convert the linear inequality in slope-intercept form. Be sure the y is on the left and remember to flip the inequality sign if you multiply or divide by a negative.
2. Graph the line as if it is an equation, except use a dotted line if the inequality sign is $<$ or $>$. If the sign is $\leq$ or $\geq$, use a regular solid line.
3. Shade above the line for a "greater than" inequality ($>$ or $\geq$). Shade below the line for a "less than" inequality ($<$ or $\leq$). (For vertical lines, shade to the right for greater than and to the left for less than).

Ex: $-3x - 2y > 8$

$$\begin{array}{rr} -3x - 2y > 8 & \\ +3x & +3x \\ \hline \end{array}$$

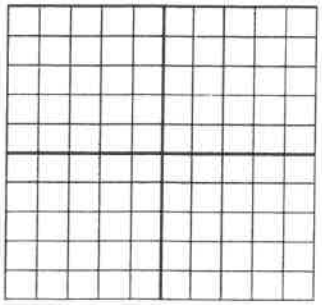
$$\frac{-2y}{-2} > \frac{3x}{-2} + \frac{8}{-2}$$

$$y < -\frac{3}{2}x - 4$$

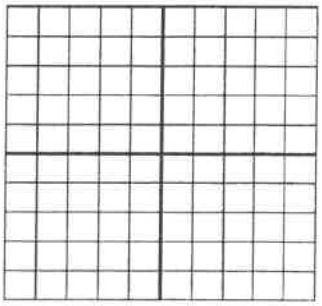


Solve each system of equations by graphing.

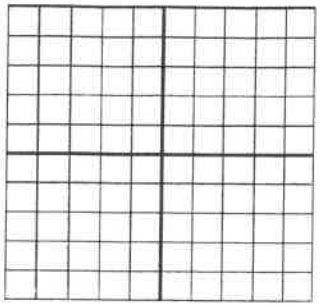
50. $\begin{cases} y = \frac{1}{2}x - 4 \\ y = -x - 1 \end{cases}$



51. $\begin{cases} y = 2x + 1 \\ -y = -2x + 1 \end{cases}$



52. $\begin{cases} x - 2y = 4 \\ -3x + 2y = -8 \end{cases}$



Solve each system of equations using substitution.

53. $\begin{cases} y = 2x + 3 \\ 5x - 2y = -6 \end{cases}$

54. $\begin{cases} x + 4y = 5 \\ -2x + 5y = 16 \end{cases}$

55. $\begin{cases} 9y - 7x = -13 \\ -9x + y = 15 \end{cases}$

Solve each system of equations using elimination.

56. $\begin{cases} 3x - 7y = -29 \\ -4x + 7y = 27 \end{cases}$

57. $\begin{cases} -4x - 8y = -48 \\ 8x + 3y = -34 \end{cases}$

58. $\begin{cases} 3x - 7y = 21 \\ 6x = 14y + 42 \end{cases}$

Solve each word problem using a system of equations.

59. Joe bought 5 apples and 4 bananas for \$6. Dawn bought 3 apples and 6 bananas for \$6.30. How much does each apple and each banana cost?

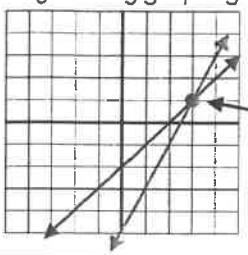
60. Wesley and Brian have a total of 87 baseball cards. Wesley has 30 less than twice as many cards as Brian. How many baseball cards do they each own?

Solving Systems of Equations by Graphing

1. Graph both lines on the same coordinate plane.
2. Find the point where the lines meet, and write that solution as an ordered pair.

Special Cases:
- parallel lines: no solution
- coincident lines (lines that are the same): infinitely many solutions

Ex: Solve the system by graphing: $\begin{cases} y = x - 2 \\ y = 2x - 5 \end{cases}$



solution: $(3, 1)$

Solving Systems of Equations Using Substitution

1. Solve one of the equations for x or y.
2. Replace the x or y in the other equation with the expression you found in step 1 that equals that variable.
3. Solve the equation.
4. Substitute the solution you found in step 3 with the variable in your step 1 equation to solve for the other variable.
5. Write your solution as an ordered pair.

Ex: Solve the system by substitution: $\begin{cases} x + 3y = 4 \\ 2x - 3y = -1 \end{cases}$

$x + 3y = 4 \rightarrow x = -3y + 4$

$2x - 3y = -1 \rightarrow 2(-3y + 4) - 3y = -1$

$\rightarrow -6y + 8 - 3y = -1$

$\rightarrow -9y + 8 = -1$

$\rightarrow -9y = -9 \rightarrow \boxed{y = 1}$

$\rightarrow x = -3y + 4 \rightarrow x = -3(1) + 4 \rightarrow \boxed{x = 1}$

solution: $\boxed{(1, 1)}$

Solving Systems of Equations Using Elimination

1. Write both equations in Standard Form.
2. Multiply neither, one, or both of the equations by constants so that either the x coefficients or the y coefficients are opposites (i.e. 2 and -2).
3. Add the two equations. The terms with the opposite coefficients will cancel out.
4. Solve the equation for the variable that didn't cancel out.
5. Substitute the solution you found in step 4 for the variable in any of the equations, and solve to find the other variable.
6. Write your solution as an ordered pair.

Ex: Solve the system by elimination: $\begin{cases} 3x + 4y = \\ -2x + 2y = -6 \end{cases}$

Multiplying by -2 will give the y terms opposite coefficients

$3x + 4y = 2$

$-2(-2x + 2y = -6)$

$\rightarrow \begin{array}{r} 3x + 4y = 2 \\ +\ 4x - 4y = 12 \\ \hline 7x \quad\ = 14 \end{array} \rightarrow \boxed{x = 2}$

$\rightarrow 3x + 4y = 2 \rightarrow 3(2) + 4y = 2$

$\rightarrow 6 + 4y = 2 \rightarrow 4y = -4 \rightarrow \boxed{y = -1}$

solution: $\boxed{(2, -1)}$

Systems of Equations Word Problems

1. Define 2 variables.
2. Write 2 equations.
3. Solve the system of equations using the method of your choice.
4. Label your solution with the appropriate units.

Ex: A 24 question test contains some 3 point questions and some 5 point questions. If the test is worth 100 points, how many of each type of questions are there?

Let x = # of 3 point questions
y = # of 5 point questions

$x + y = 24$

$3x + 5y = 100$

solve using substitution or elimination → solution: (10, 14)

→ There were 10 3-point questions and 14 5-point questions.

Simplify each expression completely. Write your answer using only positive exponents.

61. $x^6 \cdot x^4$	62. $(5^3)^2$	63. $-6a^2b^{-4}c \cdot 4ab^2$
64. $\frac{a^3b^{-6}}{c^{-2}}$	65. $\left(\frac{-2x^6y}{3z^5}\right)^3$	66. $\left(8w^3q^{-5}\right)^0$
67. $\frac{24d^5f^{-5}g^8}{36d^{-3}f^9g^2}$	68. $\left(2b^{-3}d^6\right)^4 \cdot 3b^7d$	69. $\left(\frac{-4a^4b^2c^{-1}}{6a^9}\right)^{-1}$

Exponent Rules

Zero Exponent: Any base raised to the zero power equals 1.

Ex: $(-9)^0 = \boxed{1}$

Negative Exponent: Move the base to the opposite side of the fraction bar and make the exponent positive.

Ex: $3^{-4} = \frac{1}{3^4} = \boxed{\frac{1}{81}}$

Monomial x Monomial: Multiply the coefficients and add the exponents of like bases.

Ex: $(-2x^3)(8x^{-5}) = -16x^{-2} = \boxed{\frac{-16}{x^2}}$

Monomial ÷ Monomial: Divide the coefficients and subtract the exponents of like bases.

Ex: $\frac{4ab^3}{4a^2b^2} = 1a^{-1}b^1 = \boxed{\frac{b}{a}}$

Power of a Monomial: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents.

Ex: $(3x^3y^2)^3 = 3^3x^9y^6 = \boxed{27x^9y^6}$

Power of a Quotient: Raise each base (including the coefficients) to that power. If a base already has an exponent, multiply the two exponents.

Ex: $\left(\frac{5a^3b}{2c^{-1}}\right)^2 = \frac{5^2a^6b^2}{2^2c^{-2}} = \boxed{\frac{25a^6b^2c^2}{4}}$